Analysis and Control of Functional Brain Networks

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Who really did the work


Tommaso Menara


Yuzhen Qin


Giacomo Baggio

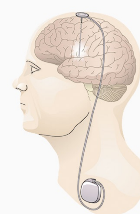
Reverse-Engineer the Human Brain

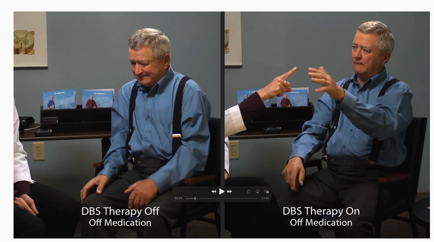
One of the greatest challenges of modern science, and the key to ...


(credit Ellen Weinstein)

Why should we reverse-engineer the brain?

- personalized therapies & reversal of cognitive decline



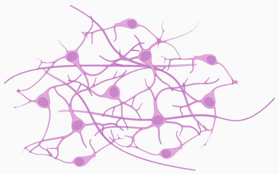


[Medtronic]

Why should we reverse-engineer the brain?

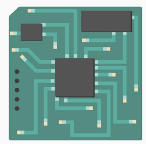
- personalized therapies & reversal of cognitive decline
- efficient computing systems & brain-computer interfaces

energy: ~ 10 W



vs

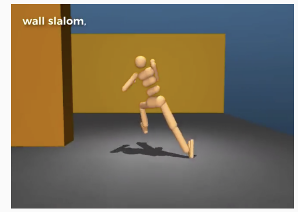
energy: ~ 100 W



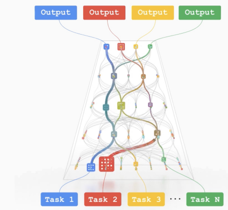
[von Neumann, 2012, The Computer & the Brain]

Why should we reverse-engineer the brain?

- personalized therapies & reversal of cognitive decline
- efficient computing systems & brain-computer interfaces
- next generation machine intelligence

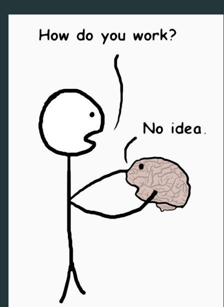


[Deepmind]

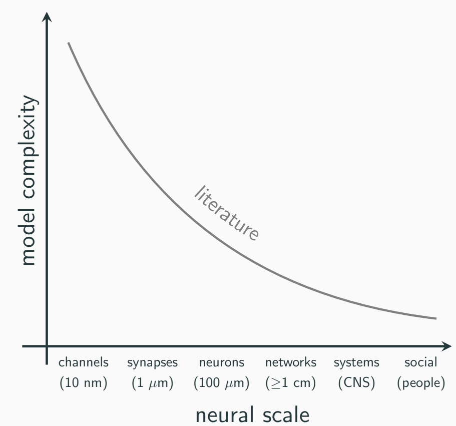


[Google Research]

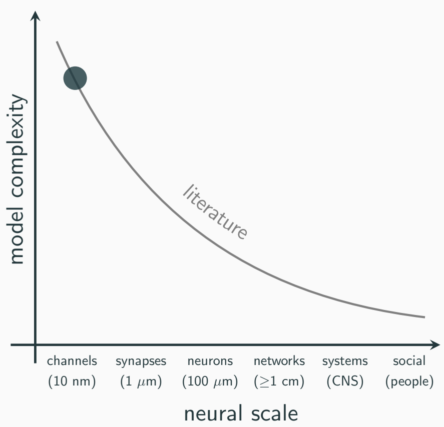
How to reverse-engineer the brain?

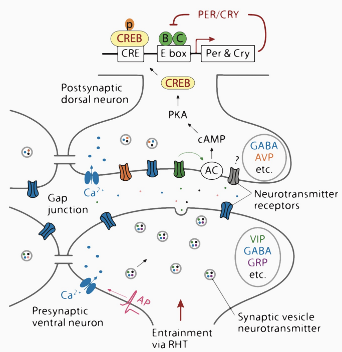


Models and scales of interest



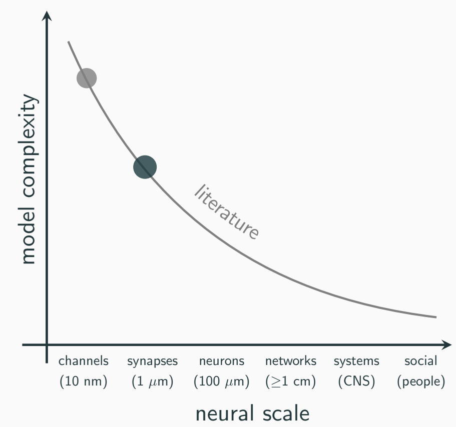
Models and scales of interest

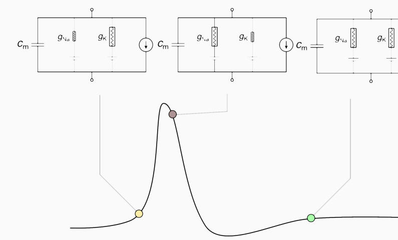




[Welsh *et al.*, 2010, Ann Rev Physiol]

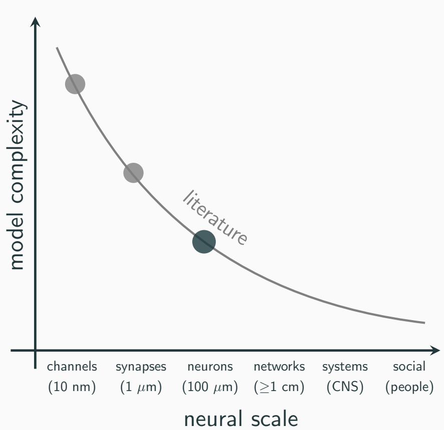
Models and scales of interest

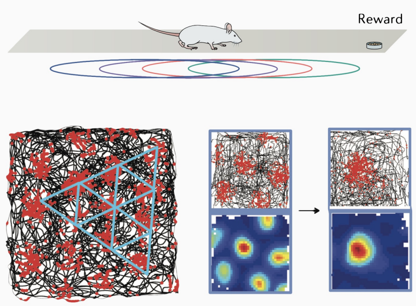




[Kumar *et al.*, 2010, Closed Loop Neuroscience]

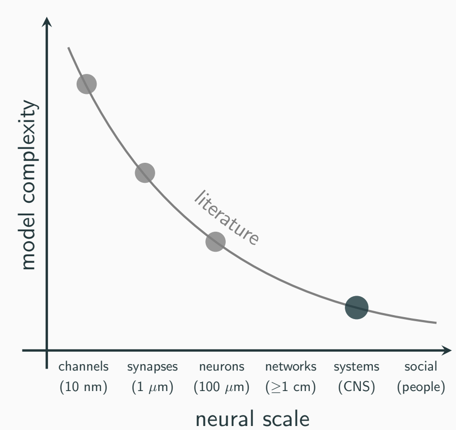
Models and scales of interest

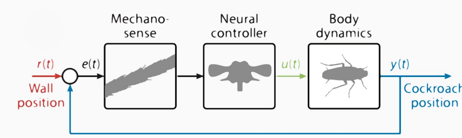




[Moser *et al.*, 2015, Cold Spring Harb Perspect Biol]

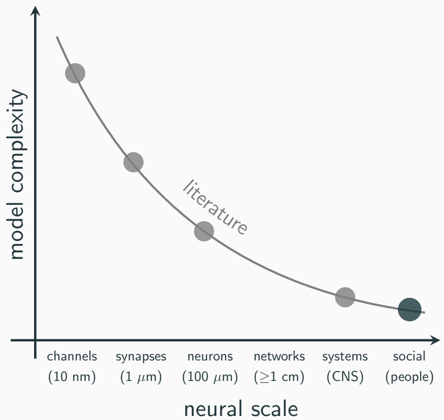
Models and scales of interest

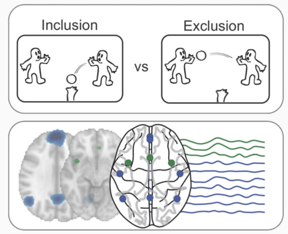




[Madhav & Cowan, 2020 Ann Rev Contr Rob Aut Sys]

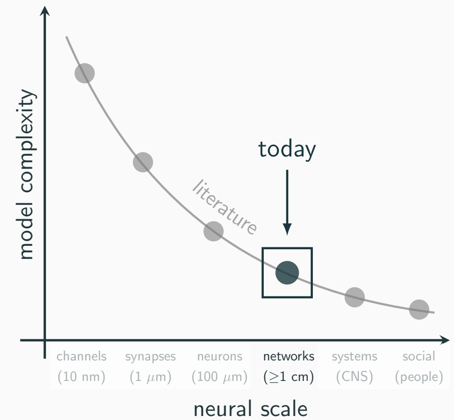
Models and scales of interest

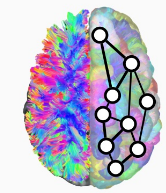




[Schmälzle *et al.*, 2017 PNAS]

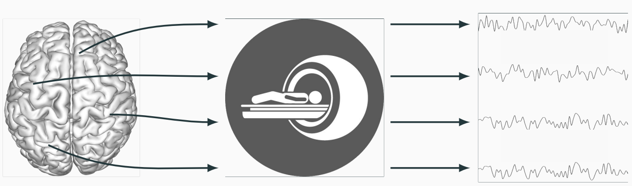
Models and scales of interest



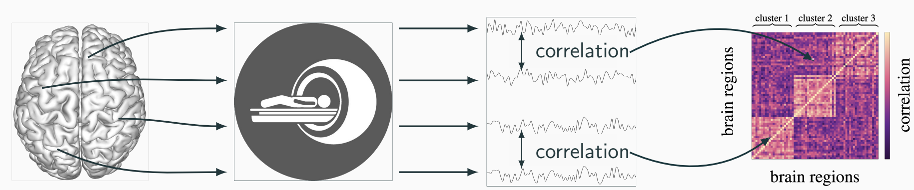


- brain-wide process (*functional patterns*)
- models retain biological compatibility
- models remain analytically tractable

Functional patterns of brain activity



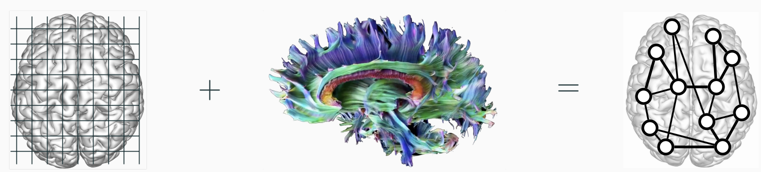
Functional patterns of brain activity



- synchrony between brain regions long known (EEG, fMRI, [Berger & Gray, 1929])
- rich repertoire of synchrony patters (transient, long-range, clustered)
- different patterns are biomarkers of health and disease (epilepsy, Parkinson's)

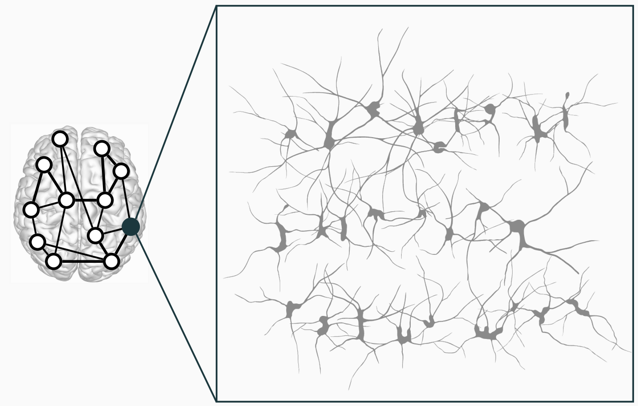
Modeling functional patterns

brain regions | white matter fibers | brain network



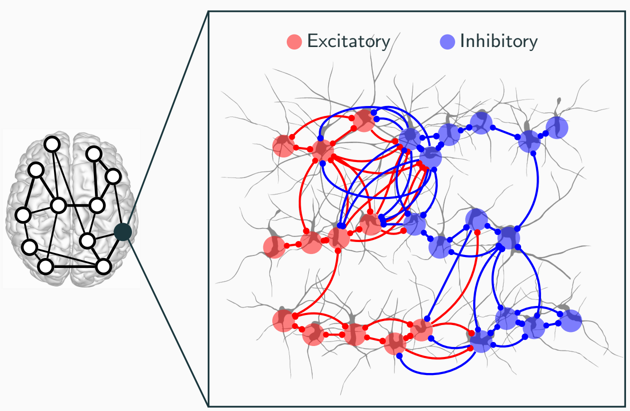
- nodes = brain regions; edges = bundles of white matter fibers
- static brain networks carry structural and statistical information
- *dynamic* brain networks are useful for the prediction & control of neural dynamics

Resting-state activity modeled by phase oscillators



each node of the brain network captures the dynamics of a population of neurons

Resting-state activity modeled by phase oscillators



excitatory and inhibitory communities are in a regime of self-sustained oscillations (weakly coupled Wilson-Cowan)
[Hoppensteadt and Izhikevich, 1997]

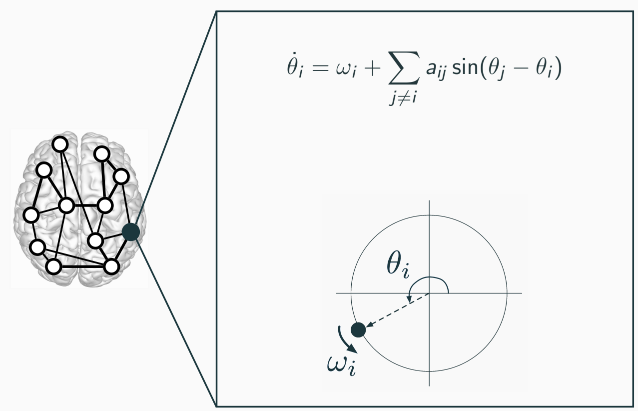
↓

neurons' firing rates describe a limit cycle

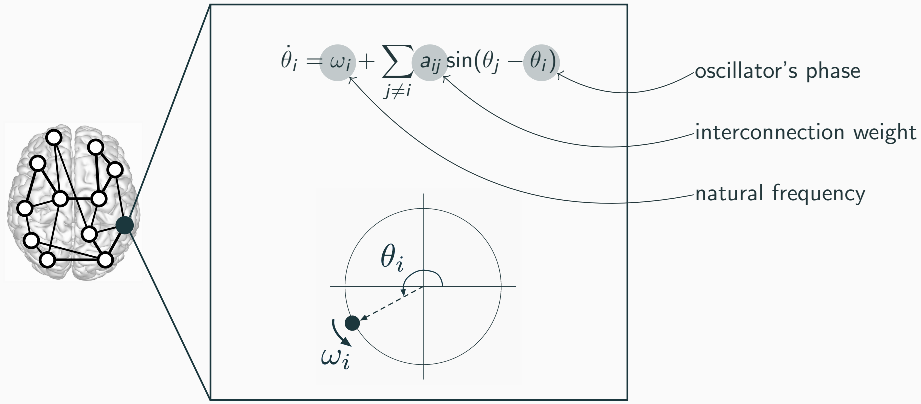
↓

dynamics approximated by a single phase variable
[Cabral *et al.*, 2011]

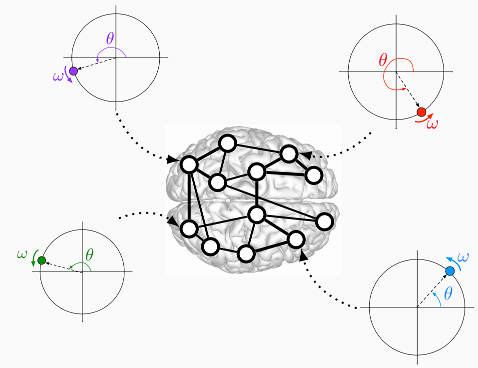
Resting-state activity modeled by phase oscillators



Resting-state activity modeled by phase oscillators



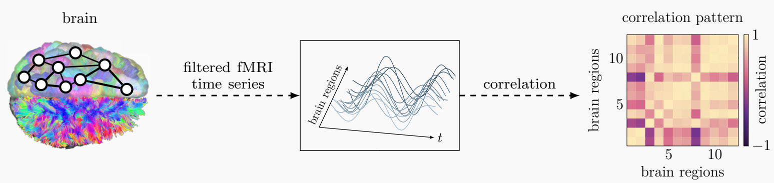
Dynamical brain network to simulate neural activity



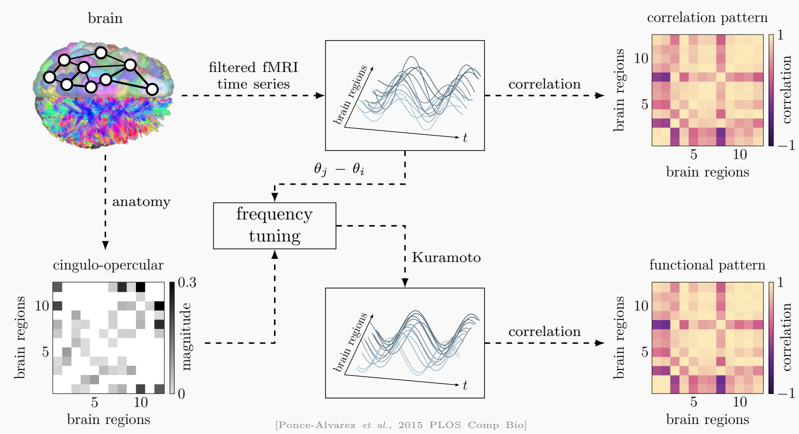
dynamical brain network with:

- nodes = brain regions
- edges = white matter fibers
- node dynamics = Kuramoto

Sanity check

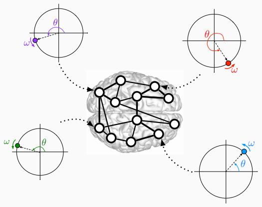


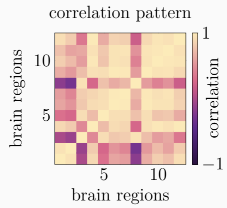
Sanity check



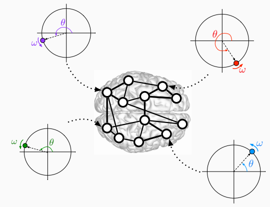
[Ponce-Alvarez *et al.*, 2015 PLOS Comp Bio]

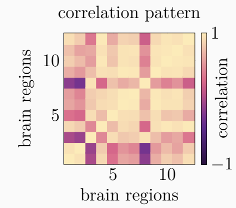
Oscillator properties and functional patterns





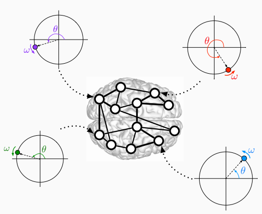
Oscillator properties and functional patterns

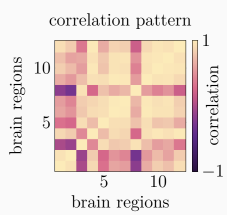




- correlated brain regions ⇔ synchronized oscillators
- correlated regions typically form disjoint clusters

Oscillator properties and functional patterns





- correlated brain regions ⇔ synchronized oscillators
- correlated regions typically form disjoint clusters

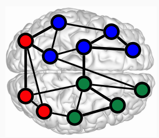
cluster synchronization in oscillator networks as a proxy for correlated neural activity

Analysis of cluster synchronization

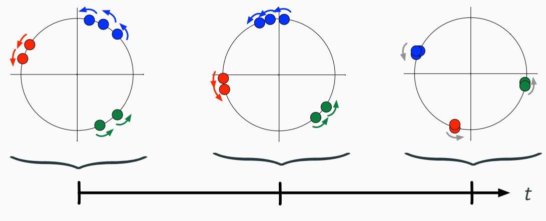
Cluster synchronization

Network partition

$\mathcal{P} = \{\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3\}$



$\Rightarrow$



Cluster synchronization

Network partition

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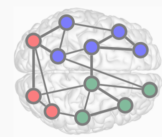
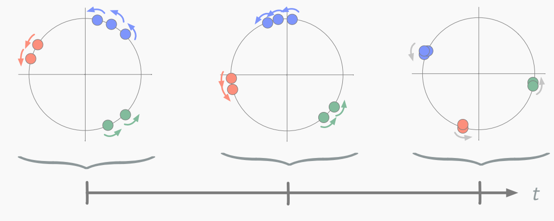


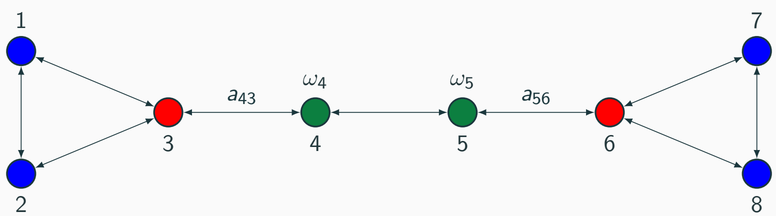
$\Rightarrow$



cluster synchronization manifold for $\mathcal{P} = \{\mathcal{C}_1, \dots, \mathcal{C}_m\}$:

$$\boxed{\mathcal{S}_{\mathcal{P}} = \{\theta \in \mathbb{T}^n \;:\; \theta_i = \theta_j \text{ for all } i, j \in \mathcal{C}_k, k = 1, \dots, m\}}$$

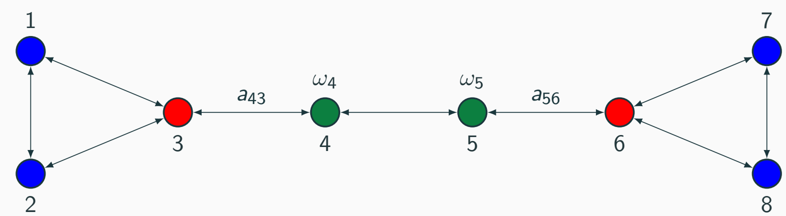
Invariance of cluster synchronization



$$\dot\theta_4 = \omega_4 + a_{43}\sin(\theta_3 - \theta_4) + a_{45}\sin(\theta_5 - \theta_4)$$

$$\dot\theta_5 = \omega_5 + a_{56}\sin(\theta_6 - \theta_5) + a_{54}\sin(\theta_4 - \theta_5)$$

Invariance of cluster synchronization



$$\dot\theta_4 - \dot\theta_5 = \underbrace{\omega_4 - \omega_5}_{\text{diff. nat. freq.}} + \underbrace{a_{43}\sin(\theta_3 - \theta_4) - a_{56}\sin(\theta_6 - \theta_5)}_{\text{diff. external coupling}}$$

oscillators 4 and 5 remain phase synchronized if:

- diff. of natural frequencies $= -$ diff. external coupling at all times
- equal natural frequencies ($\omega_4 = \omega_5$) and equal coupling ($a_{43} = a_{56}$)

Invariance of cluster synchronization

Invariance of the cluster synchronization manifold $\mathcal{S}_\mathcal{P}$ iff:

- $\omega_i = \omega_j$ for all oscillators in the same cluster
- the network weights are balanced

[Menara *et al.*, 2020 TCNS]

balanced weights for partition $\mathcal{P} = \{\mathcal{C}_1, \mathcal{C}_2, \dots\}$:

$$\sum_{z \in \mathcal{C}_\ell} a_{iz} - a_{jz} = 0 \text{ for all } i, j \in \mathcal{C}_k \text{ and all partitions } \mathcal{C}_\ell \neq \mathcal{C}_k$$

Invariance of cluster synchronization

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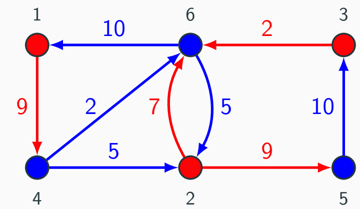
[Menara *et al.*, 2020 TCNS]



$\mathcal{C}_1 = \{1, 2, 3\}, \quad \mathcal{C}_2 = \{4, 5, 6\}$

$\mathcal{C}_1 \to \mathcal{C}_2^i = \mathcal{C}_1 \to \mathcal{C}_2^j$

$\mathcal{C}_2 \to \mathcal{C}_1^i = \mathcal{C}_2 \to \mathcal{C}_1^j$

Invariance of cluster synchronization

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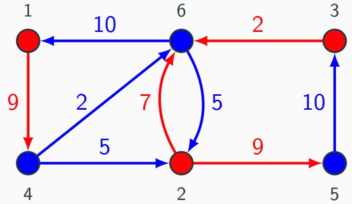
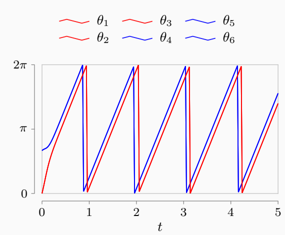
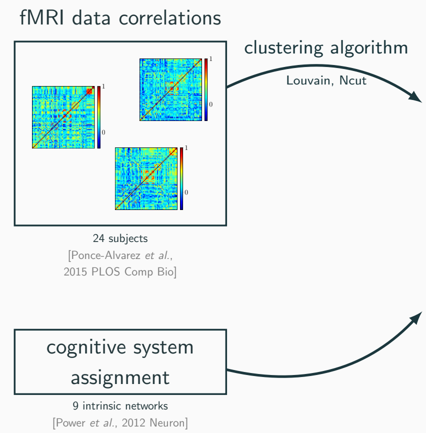


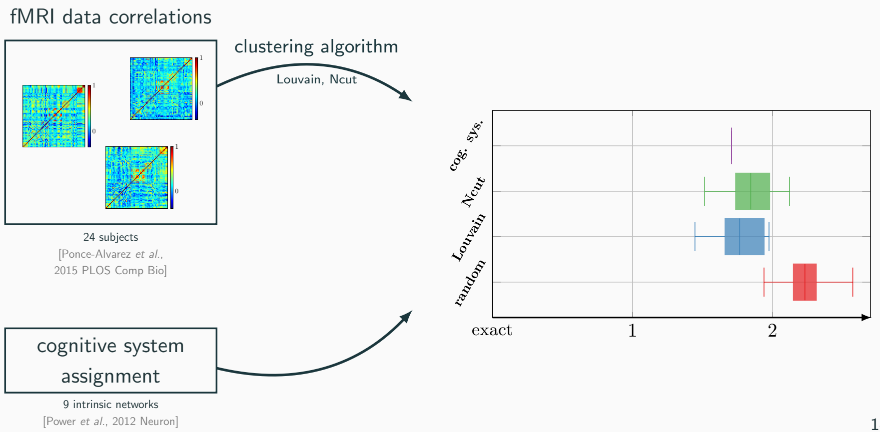


Cluster invariance in empirical brain networks

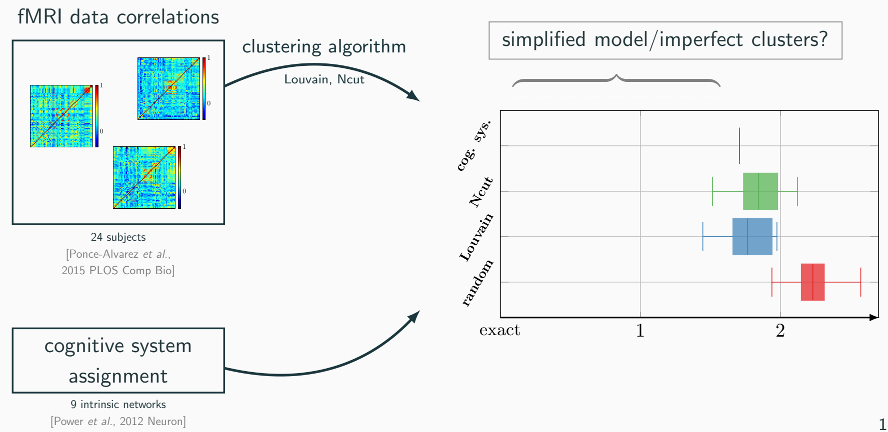


...distance from balanced network weights

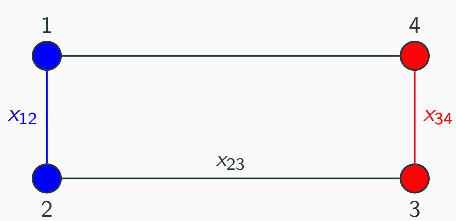
Cluster invariance in empirical brain networks



Cluster invariance in empirical brain networks



Local stability of cluster synchronization



$\mathcal{C}_2 = \{1, 2\}$

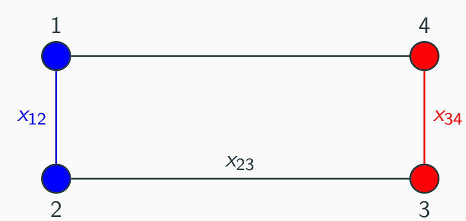
$\mathcal{C}_2 = \{3, 4\}$

$$\dot{x}_{12} = \dot{\theta}_1 - \dot{\theta}_2 = -(a_{12} + a_{21})\sin(x_{12}) + a_{13}\sin(x_{23}) - a_{13}\sin(x_{12} + x_{23} + x_{34})$$

$$\dot{x}_{34} = \dot{\theta}_3 - \dot{\theta}_4 = -(a_{34} + a_{43})\sin(x_{34}) + a_{31}\sin(x_{23}) - a_{31}\sin(x_{12} + x_{23} + x_{34})$$

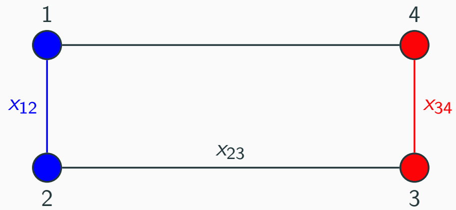
$$\dot{x}_{23} = \dot{\theta}_1 - \dot{\theta}_3 = (\omega_2 - \omega_1) + a_{43}\sin(x_{34}) + a_{21}\sin(x_{12}) - (a_{42} + a_{24})\sin(x_{23})$$

Local stability of cluster synchronization



$$\underbrace{\begin{bmatrix} \dot{x}_{12} \\ \dot{x}_{34} \end{bmatrix}}_{\dot{x}_{\text{intra}}} = \underbrace{\begin{bmatrix} -(a_{12} + a_{21})\sin(x_{12}) \\ -(a_{34} + a_{43})\sin(x_{34}) \end{bmatrix}}_{F(x_{\text{intra}})} + \underbrace{(\sin(x_{23}) - \sin(x_{12} + x_{23} + x_{34})) \begin{bmatrix} a_{13} \\ a_{31} \end{bmatrix}}_{G(x_{\text{intra}}, x_{\text{inter}})}$$

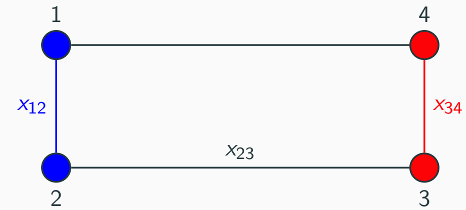
Local stability of cluster synchronization



$$\dot{x}_{\text{intra}} = F(x_{\text{intra}}) + G(x_{\text{intra}}, x_{\text{inter}})$$

- F = intra-cluster dynamics; G = inter-cluster dynamics
- the origin of F is exponentially stable with rate dep. on intra-cluster weights
  (from homogeneous Kuramoto dynamics)
- G is vanishing ($G(0, x_{\text{inter}}) = 0$) and lin. bounded ($\|G(x_{\text{intra}}, x_{\text{inter}})\| \leq \gamma \|x_{\text{intra}}\|$)

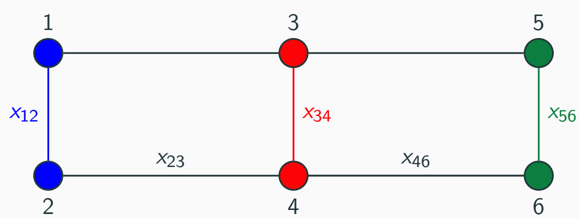
Local stability of cluster synchronization



If intra-cluster weights $\gg$ inter-cluster weights, then $x_{\text{intra}} = 0$ is locally exponentially stable, and the cluster synchronization manifold is locally exponentially stable.

[Menara *et al.*, 2020 TCNS]

Stability of multiple clusters

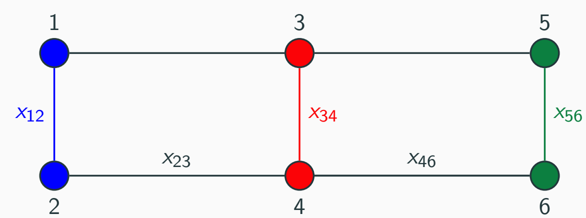


$$\dot{x}^{(k)}_{\text{intra}} = F^{(k)}(x^{(k)}_{\text{intra}}) + G^{(k)}(x_{\text{intra}}, x_{\text{inter}})$$

J_k : Jacobian of F at $x^{(k)}_{\text{intra}} = 0$ $\qquad$ X_k : solution to $J_k^{\mathsf{T}} X_K + X_k J_k = -I$

$$\gamma^{(k\ell)} : \|G^{(k)}(x_{\text{intra}}, x_{\text{inter}})\| \leq \textstyle\sum_{\ell=1}^{m} \gamma^{(k\ell)} \|x^{(\ell)}_{\text{intra}}\|$$

Stability of multiple clusters



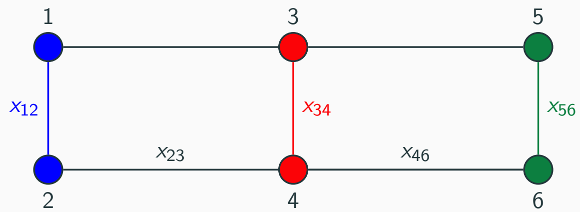
If the following matrix is an M-matrix,

$$S = [s_{k\ell}] = \begin{cases} \lambda_{\max}^{-1}(X_k) - \gamma^{(kk)} & \text{if } k = \ell, \\ -\gamma^{(k\ell)} & \text{if } k \neq \ell, \end{cases}$$

then the synchronization manifold is locally exponentially stable.

[Menara *et al.*, 2020 TCNS]

Stability of multiple clusters



If the following matrix is an M-matrix,

intra-cluster weights $\gg$ inter-cluster weights $\Rightarrow$ stability

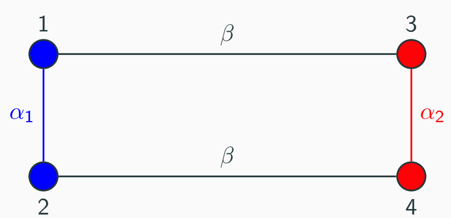
then the synchronization manifold is locally exponentially stable.

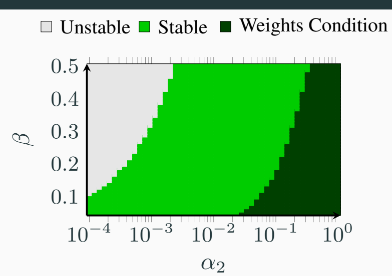
[Menara *et al.*, 2020 TCNS]

An example

- fix α_1, ω_1, ω_2 and vary α_2, β

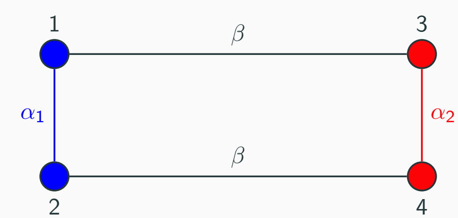
An example

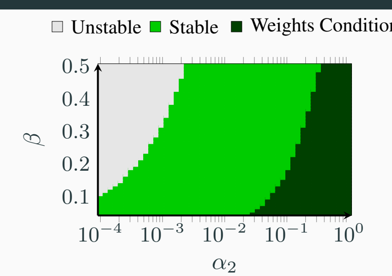




- fix α_1, ω_1, ω_2 and vary α_2, β

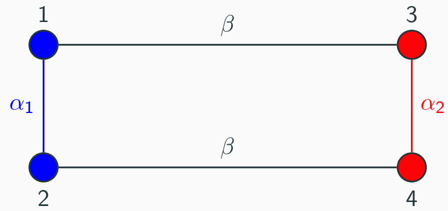
An example

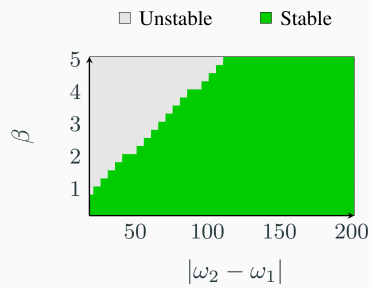




- fix α_1, ω_1, ω_2 and vary α_2, β
- network weights provide conservative estimates of stability. frequencies?

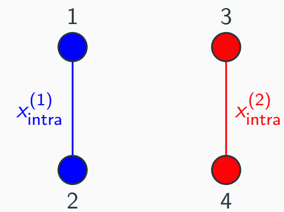
An example





- fix α_1, α_2, and vary β, $|\omega_2 - \omega_1|$
- network weights provide conservative estimates of stability. frequencies?
- large frequency differences promote stability. why?

Approximate stability of cluster synchronization

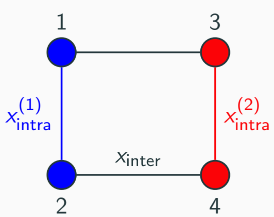


- isolated clusters are stable oscillatory systems
- linearized dynamics:

$$\dot{x}^{(1)}_{\text{intra}} = J_1 x^{(1)}_{\text{intra}}$$

- connectivity + homogeneous frequencies $\Rightarrow$ intra-cluster synchronization

Approximate stability of cluster synchronization

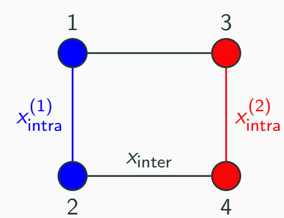


- isolated clusters are stable oscillatory systems
- clusters subject to inter-cluster perturbation
- linearized dynamics:

$$\dot{x}^{(1)}_{\text{intra}} = J_1 x^{(1)}_{\text{intra}} + \eta_{12} \, \mathbf{cos}(x_{\text{inter}}) \, x^{(2)}_{\text{intra}}$$

- $x_{\text{inter}} \to (\omega_2 - \omega_1)t$ as $|\omega_2 - \omega_1|$ grows

Approximate stability of cluster synchronization

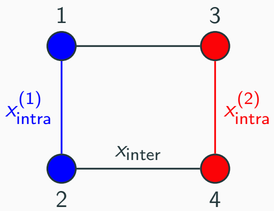


- isolated clusters are stable oscillatory systems
- clusters subject to inter-cluster perturbation
- linearized dynamics:

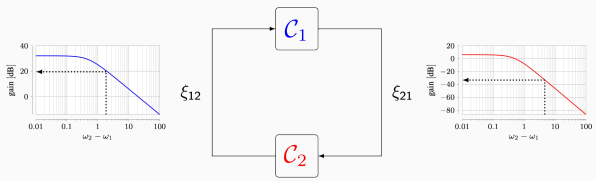
$$\dot{x}^{(1)}_{\text{intra}} \approx J_1 x^{(1)}_{\text{intra}} + \eta_{12} \, \mathbf{cos}((\omega_2 - \omega_1)t) \, x^{(2)}_{\text{intra}}$$

- inter-cluster perturbation is modulated by $\omega_2 - \omega_1$

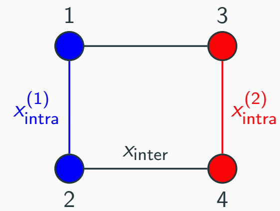
Approximate stability of cluster synchronization



- isolated clusters are stable oscillatory systems
- clusters subject to inter-cluster perturbation
- isolated clusters behave as **low-pass filters**



Approximate stability of cluster synchronization

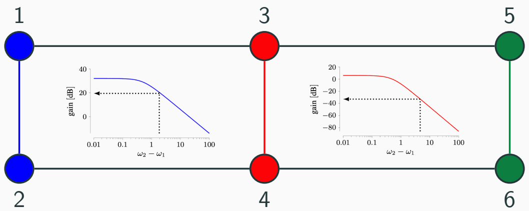


- isolated clusters are stable oscillatory systems
- clusters subject to inter-cluster perturbation
- isolated clusters behave as **low-pass filters**

If $|\omega_j - \omega_i| \to \infty$ for all clusters, then the synchronization manifold is locally exponentially stable.

[Menara *et al.*, 2020 TCNS]

Stability of multiple clusters using weights and frequencies

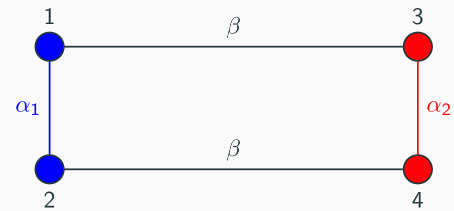


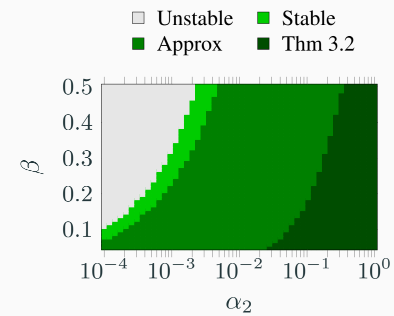
$\xi_{ij} = |$ frequency gain from cluster $\mathcal{C}_i$ to cluster $\mathcal{C}_j|$

If $\lambda_{\max}([\xi_{ij}]) < 1$, then the sync. manifold is locally exp. stable.

[Menara *et al.*, 2019 ACC]

An example

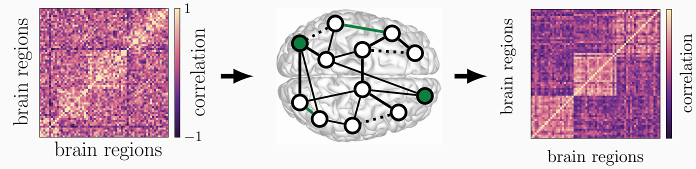




- fix α_1, ω_1, ω_2 and vary α_2, β
- combine weights and frequency gains into small-gain condition
- combined weights/frequency conditions are consistently tight

Control of cluster synchronization

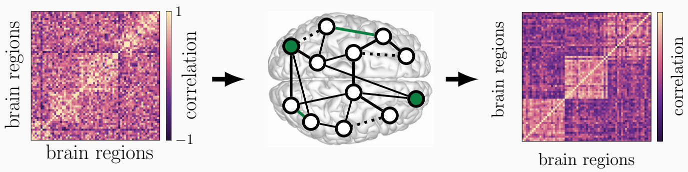
Control of functional patterns



so far...

- modeling of neural activity through oscillator network
- modeling of functional patterns via cluster synchronization
- conditions for invariance/stability of cluster synchronization

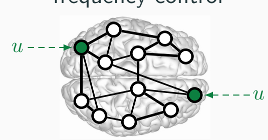
Control of functional patterns



how do we restore healthy functional patterns?

Control of functional patterns

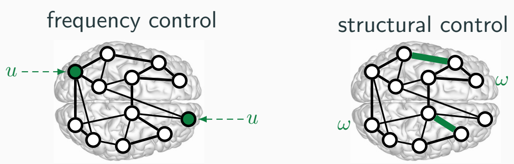
frequency control



- external control of oscillator frequency

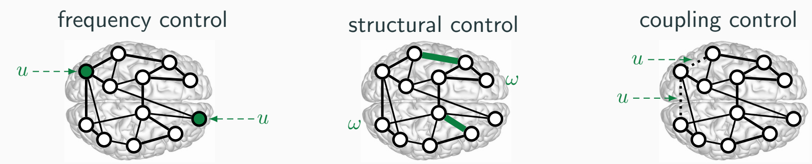
[Menara *et al.*, 2020 LCSS]

Control of functional patterns



- external control of oscillator frequency [Menara *et al.*, 2020 LCSS]
- design of structural weights and oscillator frequencies [Menara *et al.*, 2019 CDC & 2022 NatComm]

Control of functional patterns



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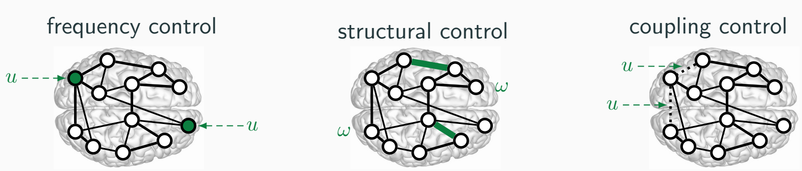
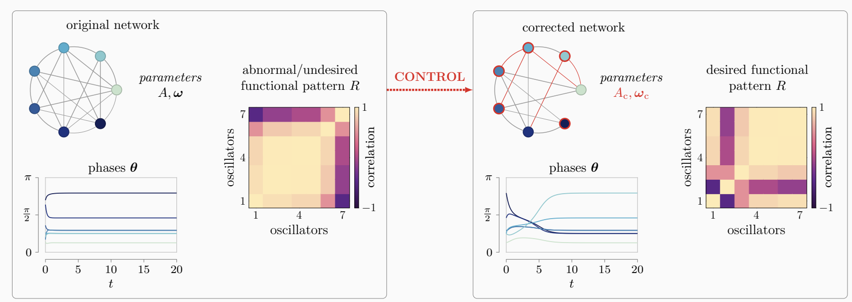
- external control of oscillators coupling [Qin *et al.*, 2022 CDC]

Control of functional patterns



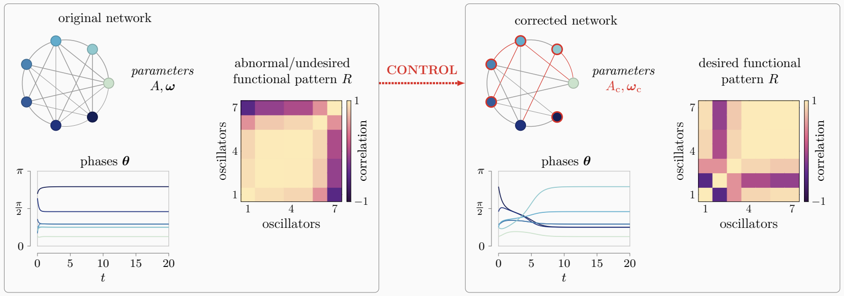
- external control of oscillator frequency [Menara *et al.*, 2020 LCSS]
- design of structural weights and oscillator frequencies [Menara *et al.*, 2019 CDC & 2022 NatComm]
- external control of oscillators coupling [Qin *et al.*, 2022 CDC]

Structural control of functional patterns



- control knobs = network weights + oscillator frequencies
- biological constraints: positive weights, sparsity of interventions
- reference signal is $n \times n$ matrix of the phase correlation values (time-varying)

Structural control of functional patterns



- control knobs = network weights + oscillator frequencies
- biological constraints: positive weights, sparsity of interventions
- reference signal is $n \times n$ matrix of the phase correlation values (time-varying)

focus on time-invariant patterns, equilibrium assignment

Frequency-synchronization and functional patterns

network dynamics in matrix form (B = incidence matrix):

$$\begin{bmatrix} \dot\theta_1 \\ \vdots \\ \dot\theta_n \end{bmatrix} = \begin{bmatrix} \omega_1 \\ \vdots \\ \omega_n \end{bmatrix} - B \begin{bmatrix} \ddots & & \\ & \sin(\theta_j - \theta_i) & \\ & & \ddots \end{bmatrix} \begin{bmatrix} \vdots \\ a_{ij} \\ \vdots \end{bmatrix}$$

Frequency-synchronization and functional patterns

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$$\begin{bmatrix} \dot\theta_1 \\ \vdots \\ \dot\theta_n \end{bmatrix} = \begin{bmatrix} \omega_1 \\ \vdots \\ \omega_n \end{bmatrix} - B \begin{bmatrix} \ddots & & \\ & \sin(\theta_j - \theta_i) & \\ & & \ddots \end{bmatrix} \begin{bmatrix} \vdots \\ a_{ij} \\ \vdots \end{bmatrix}$$

when oscillators are frequency-synchronized:

- oscillator frequencies are all equal to $\omega_{\text{mean}} = \frac{1}{n}\sum \omega_i$
- functional correlations are defined by phase differences
- feasible functional patterns have only $n-1$ degrees of freedom

Frequency-synchronization and functional patterns

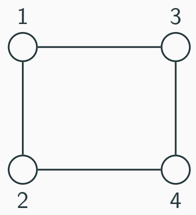
frequency-synchronized configuration:

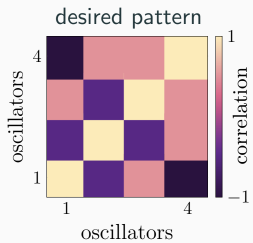
$$B \begin{bmatrix} \ddots & & \\ & \sin(\theta_j - \theta_i) & \\ & & \ddots \end{bmatrix} \begin{bmatrix} \vdots \\ a_{ij} \\ \vdots \end{bmatrix} = \begin{bmatrix} \omega_1 - \omega_{\text{mean}} \\ \vdots \\ \omega_n - \omega_{\text{mean}} \end{bmatrix}$$

to generate a desired functional pattern:

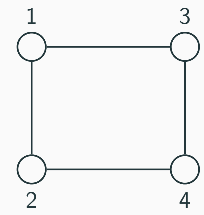
- compute $n-1$ phase differences corresponding to desired functional values
- determine feasibility of the desired equilibrium (sign/sparsity constraints)
- find network weights and frequencies to satisfy the above equation

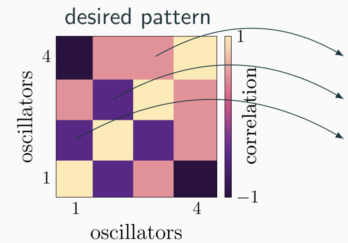
Feasibility of a functional pattern





Feasibility of a functional pattern



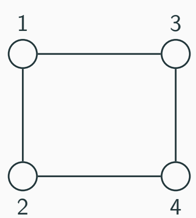


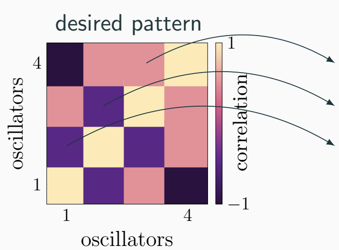
$\cos(\theta_4-\theta_3)=0.5$

$\cos(\theta_3-\theta_2)=0.4$

$\cos(\theta_2-\theta_1)=0.3$

Feasibility of a functional pattern





$\cos(\theta_4-\theta_3)=0.5$

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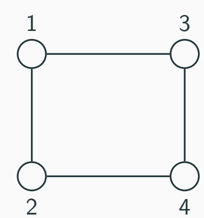
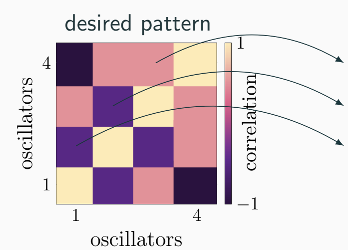
$\cos(\theta_2-\theta_1)=0.3$

$$\begin{bmatrix}-1&0&0&1\\1&-1&0&0\\0&1&-1&0\\0&0&1&-1\end{bmatrix}\begin{bmatrix}\sin(\theta_2-\theta_1)&&&\\&\sin(\theta_3-\theta_2)&&\\&&\sin(\theta_4-\theta_3)&\\&&&\sin(\theta_1-\theta_4)\end{bmatrix}\begin{bmatrix}a_{12}\\a_{23}\\a_{34}\\a_{41}\end{bmatrix}=\begin{bmatrix}\omega_1-\omega_{\text{mean}}\\\omega_2-\omega_{\text{mean}}\\\omega_3-\omega_{\text{mean}}\\\omega_4-\omega_{\text{mean}}\end{bmatrix}$$

Feasibility of a functional pattern





$\cos(\theta_4-\theta_3)=0.5$

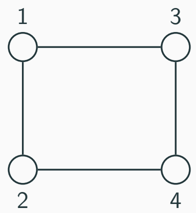
$\cos(\theta_3-\theta_2)=0.4$

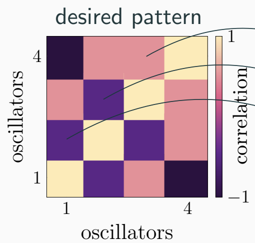
$\cos(\theta_2-\theta_1)=0.3$

given

$$\begin{bmatrix}-1&0&0&1\\1&-1&0&0\\0&1&-1&0\\0&0&1&-1\end{bmatrix}\begin{bmatrix}\sin(\theta_2-\theta_1)&&&\\&\sin(\theta_3-\theta_2)&&\\&&\sin(\theta_4-\theta_3)&\\&&&\sin(\theta_1-\theta_4)\end{bmatrix}\begin{bmatrix}a_{12}\\a_{23}\\a_{34}\\a_{41}\end{bmatrix}=\begin{bmatrix}\omega_1-\omega_{\text{mean}}\\\omega_2-\omega_{\text{mean}}\\\omega_3-\omega_{\text{mean}}\\\omega_4-\omega_{\text{mean}}\end{bmatrix}$$

Feasibility of a functional pattern





$$\cos(\theta_4-\theta_3)=0.5$$
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?

$$\begin{bmatrix}-1&0&0&1\\1&-1&0&0\\0&1&-1&0\\0&0&1&-1\end{bmatrix}\begin{bmatrix}\sin(\theta_2-\theta_1)&&&\\&\sin(\theta_3-\theta_2)&&\\&&\sin(\theta_4-\theta_3)&\\&&&\sin(\theta_1-\theta_4)\end{bmatrix}\begin{bmatrix}a_{12}\\a_{23}\\a_{34}\\a_{41}\end{bmatrix}=\begin{bmatrix}\omega_1-\omega_{\text{mean}}\\\omega_2-\omega_{\text{mean}}\\\omega_3-\omega_{\text{mean}}\\\omega_4-\omega_{\text{mean}}\end{bmatrix}$$

Feasibility of a functional pattern with positive weights

$$\begin{bmatrix}-1&0&0&1\\1&-1&0&0\\0&1&-1&0\\0&0&1&-1\end{bmatrix}\begin{bmatrix}\sin(\theta_2-\theta_1)&&&\\&\sin(\theta_3-\theta_2)&&\\&&\sin(\theta_4-\theta_3)&\\&&&\sin(\theta_1-\theta_4)\end{bmatrix}\begin{bmatrix}a_{12}\\a_{23}\\a_{34}\\a_{41}\end{bmatrix}=\begin{bmatrix}\omega_1-\omega_{\text{mean}}\\\omega_2-\omega_{\text{mean}}\\\omega_3-\omega_{\text{mean}}\\\omega_4-\omega_{\text{mean}}\end{bmatrix}$$

Feasibility of a functional pattern with positive weights

scaled incidence matrix $\bar{B}$

$$\begin{bmatrix}-1&0&0&1\\1&-1&0&0\\0&1&-1&0\\0&0&1&-1\end{bmatrix}\begin{bmatrix}\sin(\theta_2-\theta_1)&&&\\&\sin(\theta_3-\theta_2)&&\\&&\sin(\theta_4-\theta_3)&\\&&&\sin(\theta_1-\theta_4)\end{bmatrix}\begin{bmatrix}a_{12}\\a_{23}\\a_{34}\\a_{41}\end{bmatrix}=\begin{bmatrix}\omega_1-\omega_{\text{mean}}\\\omega_2-\omega_{\text{mean}}\\\omega_3-\omega_{\text{mean}}\\\omega_4-\omega_{\text{mean}}\end{bmatrix}$$

Feasibility of a functional pattern with positive weights

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$$\begin{bmatrix}-1&0&0&1\\1&-1&0&0\\0&1&-1&0\\0&0&1&-1\end{bmatrix}\begin{bmatrix}\sin(\theta_2-\theta_1)&&&\\&\sin(\theta_3-\theta_2)&&\\&&\sin(\theta_4-\theta_3)&\\&&&\sin(\theta_1-\theta_4)\end{bmatrix}\begin{bmatrix}a_{12}\\a_{23}\\a_{34}\\a_{41}\end{bmatrix}=\begin{bmatrix}\omega_1-\omega_{\text{mean}}\\\omega_2-\omega_{\text{mean}}\\\omega_3-\omega_{\text{mean}}\\\omega_4-\omega_{\text{mean}}\end{bmatrix}$$

The functional pattern is feasible with pos. weights if:

- the network $\bar{B}$ contains a Hamiltonian path $\mathcal{H}$
- $\omega^\mathsf{T}\bar{B}_{\mathcal{H}} > 0$

[Menara *et al.*, 2022 NatComm]

Feasibility of a functional pattern with positive weights

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$$\begin{bmatrix}-1&0&0&1\\1&-1&0&0\\0&1&-1&0\\0&0&1&-1\end{bmatrix}\begin{bmatrix}\sin(\theta_2-\theta_1)&&&\\&\sin(\theta_3-\theta_2)&&\\&&\sin(\theta_4-\theta_3)&\\&&&\sin(\theta_1-\theta_4)\end{bmatrix}\begin{bmatrix}a_{12}\\a_{23}\\a_{34}\\a_{41}\end{bmatrix}=\begin{bmatrix}\omega_1-\omega_{\text{mean}}\\\omega_2-\omega_{\text{mean}}\\\omega_3-\omega_{\text{mean}}\\\omega_4-\omega_{\text{mean}}\end{bmatrix}$$

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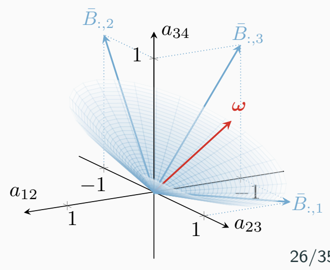

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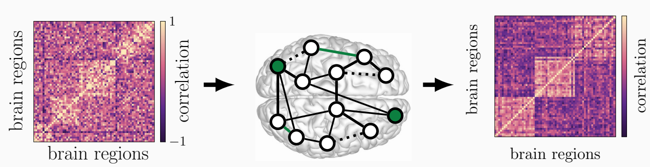
[Menara *et al.*, 2022 NatComm]

If pattern is feasible, solve (convex):

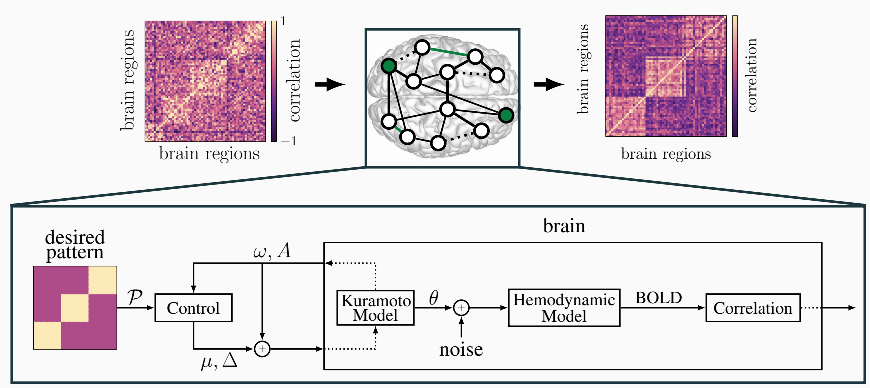
min $J(a)$
s.t. $a > 0$ and $\bar{B}a = \omega$

Restoring functional connectivity in the damaged brain



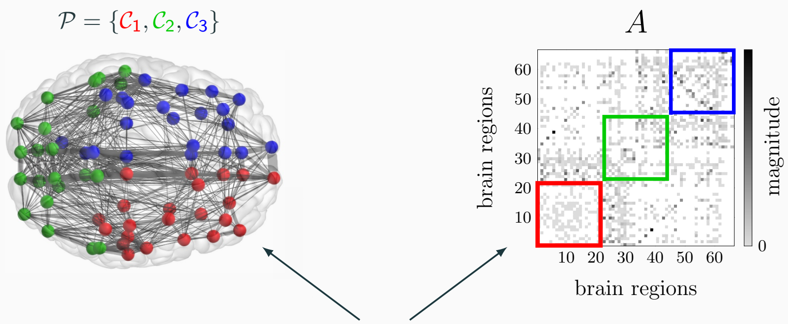
- structural data from Human Connectome Project
- (synthetic) functional data inspired by brain injury
- Balloon-Windkessel hemodynamic model for BOLD signals

Restoring functional connectivity in the damaged brain



Restoring functional connectivity in the damaged brain

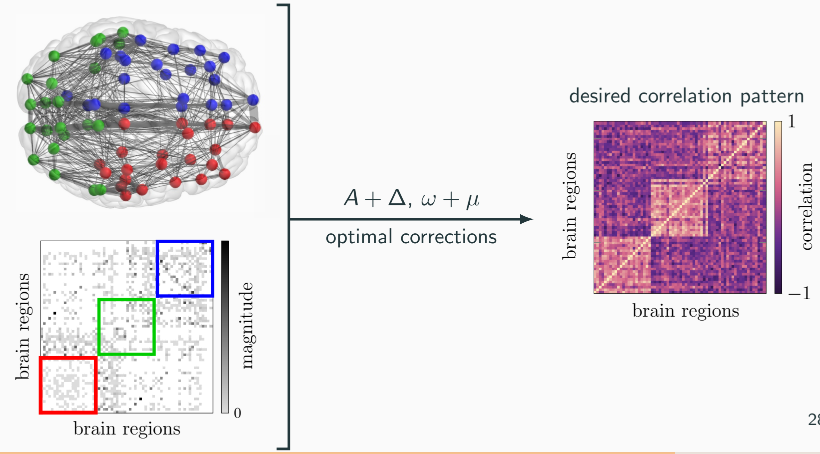
$\mathcal{P} = \{\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3\}$



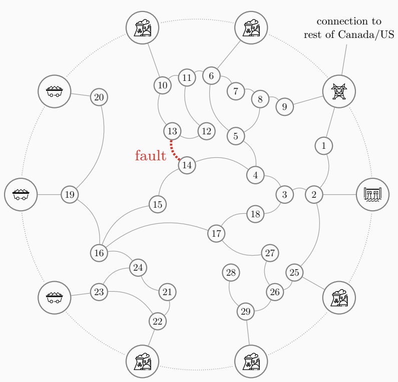
damaged

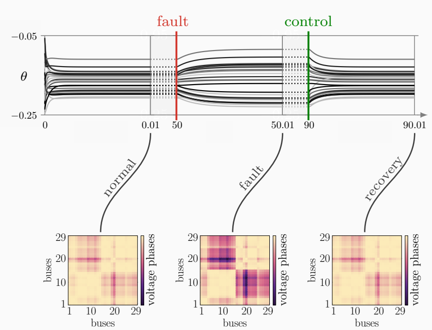
(brain injury, multiple sclerosis, ...)

Restoring functional connectivity in the damaged brain



Beyond brain networks: power redistribution and fault recovery





[Menara *et al.*, 2022 NatComm]

Summary

Modeling, analysis, control of functional connectivity via cluster synchronization:

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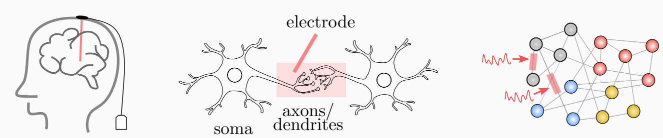
Control:	**graph-theoretic cond. for feasibility of functional patterns**
	structural control of functional patterns in brain/power

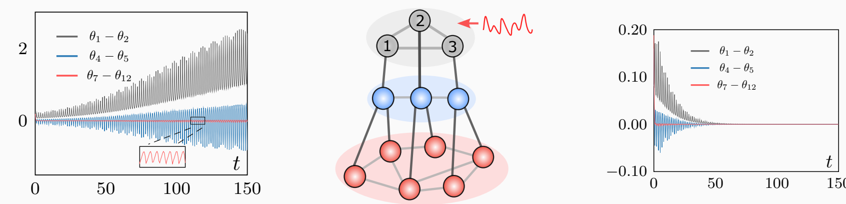
External control of oscillators coupling



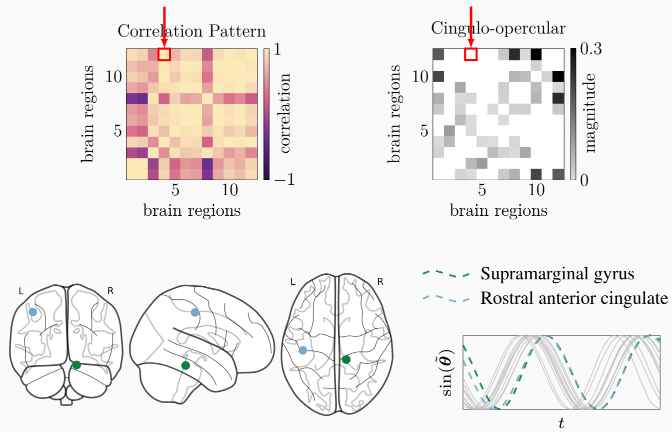
DBS

↕

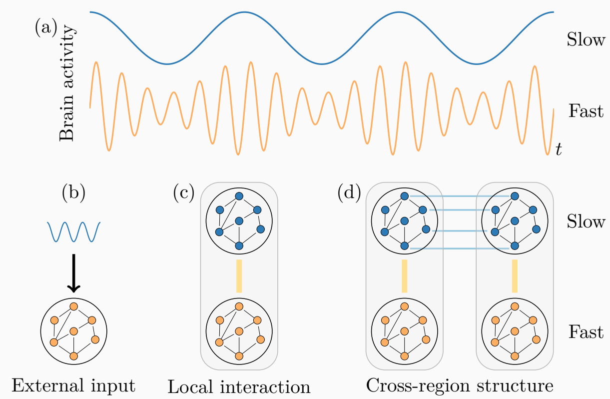
vibrational control of network edges



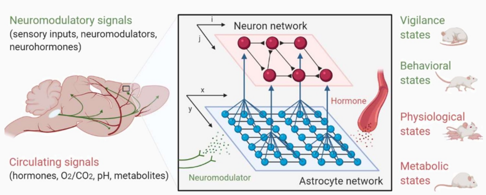
Remote (long-range) synchronization

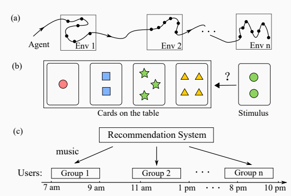


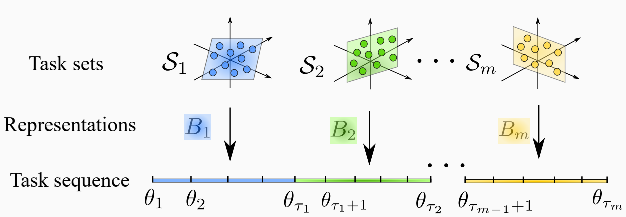
Phase-amplitude synchronization



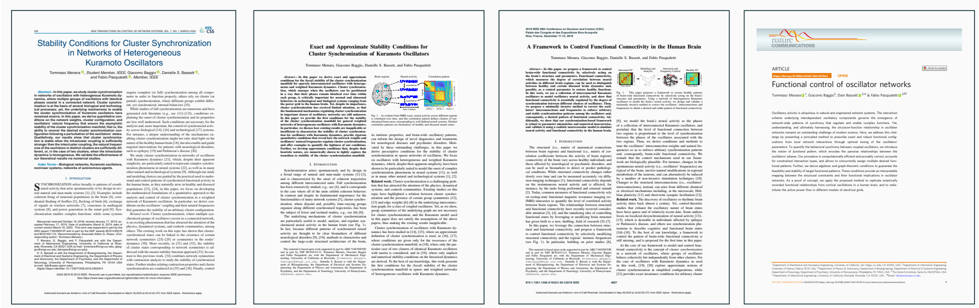
Brain-inspired, context-aware reinforcement learning







References and acknowledgements



UNIVERSITY OF CALIFORNIA RESEARCH INITIATIVES

asap Aligning Science Across Parkinson's

Analysis and Control of Functional Brain Networks

Fabio Pasqualetti

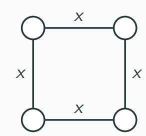
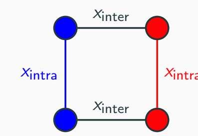
October 16, 2023

Department of Mechanical Engineering
University of California at Riverside

UC RIVERSIDE

Full vs cluster synchronization

Phase differences: $x_{ij} = \theta_j - \theta_i$

full synchronization: $x \to 0$	**cluster** synchronization: $x_{\text{intra}} \to 0$, $x_{\text{inter}} = ?$
	

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◇ Linearization around synchronized trajectory $\dot{x} = Jx$ → known ☺	◇ Linearization around cluster-synchronized trajectory $\dot{x}_{\text{intra}} = (J_{\text{intra}} + J_{\text{inter}}(t))x_{\text{intra}}$

cluster-synchronized trajectory → **unknown** ☹

J_{intra} → Hurwitz ☺

$J_{\text{inter}}(t)$ → **time-varying** ☹